Verifying Protocols with F*

Karthikeyan Bhargavan

MPRI 2:30, September 2022

Modeling Crypto and Protocol Execution

A symbolic model of bitstrings

```
type bytes =
    | Constant: string → bytes
    | Fresh: n:ℕ → bytes
    | Concat: bytes → bytes → bytes
    | AEnc: k:bytes → r:bytes → p:bytes → bytes
    | PK: bytes → bytes
    | PEnc: bytes → bytes → bytes
    | VK: bytes → bytes
    | Sig: bytes → bytes → bytes
```

A symbolic model of crypto

```
let pke_enc pk m = PEnc pk m
let pke_dec sk c =
  match c with
  | PEnc p m → if p = PK sk then Some m else None
  | _ → None


let sign sk m = Sig sk m
let verify vk m sg =
  match sg with
  | Sig sk m' → if vk = VK sk && m = m' then true else false
  | _ → false
```

A global protocol trace

```
type principal = string

noeq type entry =
      | FreshGen: p:principal → entry
      | Send: from:principal → to:principal → msg:bytes → entry
      | Store: at:principal → state:bytes → entry
      | Event: p:principal → ev:bytes → entry
      | Compromise: p:principal → entry

type trace = list entry
```

Executing Protocol Actions (1)

```
let recv p : trace → option bytes =
  let rec recv_aux p tr : option bytes =
    match tr with
    | [] → None
    | Send from to msg::tr' → if to = p then Some msg
                                        else recv_aux p tr'
    | _ :: tr' → recv_aux p tr'
  in
  recv_aux p

let retrieve p : trace → option bytes =
```

Executing Attacker Actions

```
let compromise p : trace → trace =
    λ tr → Compromise p :: tr
```

- Attacker can call **compromise p** to gain control of **p**
- Attacker can call **gen p** (for compromised **p**) to get fresh bytes
- Attacker can call **recv p** (to read any message)
- Attacker can call **retrieve p** (for compromised **p**) to read its state
- Attacker can call **send p1 p2 m** (for any m it *knows*)
- Attacker cannot call **trigger** or **store**

Attacker Knowledge

$$\textbf{val}\ \texttt{attacker\_knows:}\ \texttt{trace} \rightarrow \texttt{bytes} \rightarrow \texttt{Type}_0$$

- Attacker always knows **Constant s**
- Attacker learns **msg** from each **Send from to msg** in trace
- Attacker learns **st** from each **Store p st** (for compromised **p**)
- Attacker can call any crypto function with values it already knows:
  concat, split, ae_enc, ae_dec, pk_enc, pk_dec, sign, hash, ...

```
type bytes =
  | Constant: string → bytes
  | Fresh: n:ℕ → bytes
  | Concat: bytes → bytes → bytes
  | AEnc: k:bytes → r:bytes → p:bytes → bytes
  | PK: bytes → bytes
  | PEnc: bytes → bytes → bytes
  | VK: bytes → bytes
  | Sig: bytes → bytes → bytes
```

```
type entry =
  | FreshGen: p:principal → entry
  | Send: from:principal → to:principal → msg:bytes → entry
  | Store: at:principal → state:bytes → entry
  | Event: p:principal → ev:bytes → entry
  | Compromise: p:principal → entry
```

Reachable Traces

```
(* Some Protocol Code *)
val sendMsg1: principal → principal → trace → trace
val recvMsg1: principal → trace → trace
```

```
(* Reachability *)
let rec reachable (tr:trace) : Type =
 (∃ p₁ p₂ tr'. tr == sendMsg1 p₁ p₂ tr') ⋀ reachable tr') ⋁
 (∃ p tr'. tr == recvMsg1 p tr') ⋀ reachable tr') ⋁
 (match tr with
  | [] → ⊤
  | FreshGen p::tr' → List.mem (Compromise p) tr' ∧ reachable tr'
  | Send p₁ p₂ m::tr' → attacker_knows tr' m ∧ reachable tr'
  | Compromise p::tr' → reachable tr'
  | _ → ⊥)
```

Stating Secrecy Goals

```
let protocol_sent p secret tr =
  List.mem (Event p (concat (literal "Send") secret)) tr

let compromised p tr =
  List.mem (Compromise p) tr

let secrecy_lemma ():
  Lemma (∀ tr p m. (reachable tr ∧
                    protocol_sent p m tr ∧
                    attacker_knows tr m) ⟹
                    compromised p tr) =
```

- Proof by induction on the length of the trace
- Case analysis on all reachable traces
- Reason about possible attacker actions
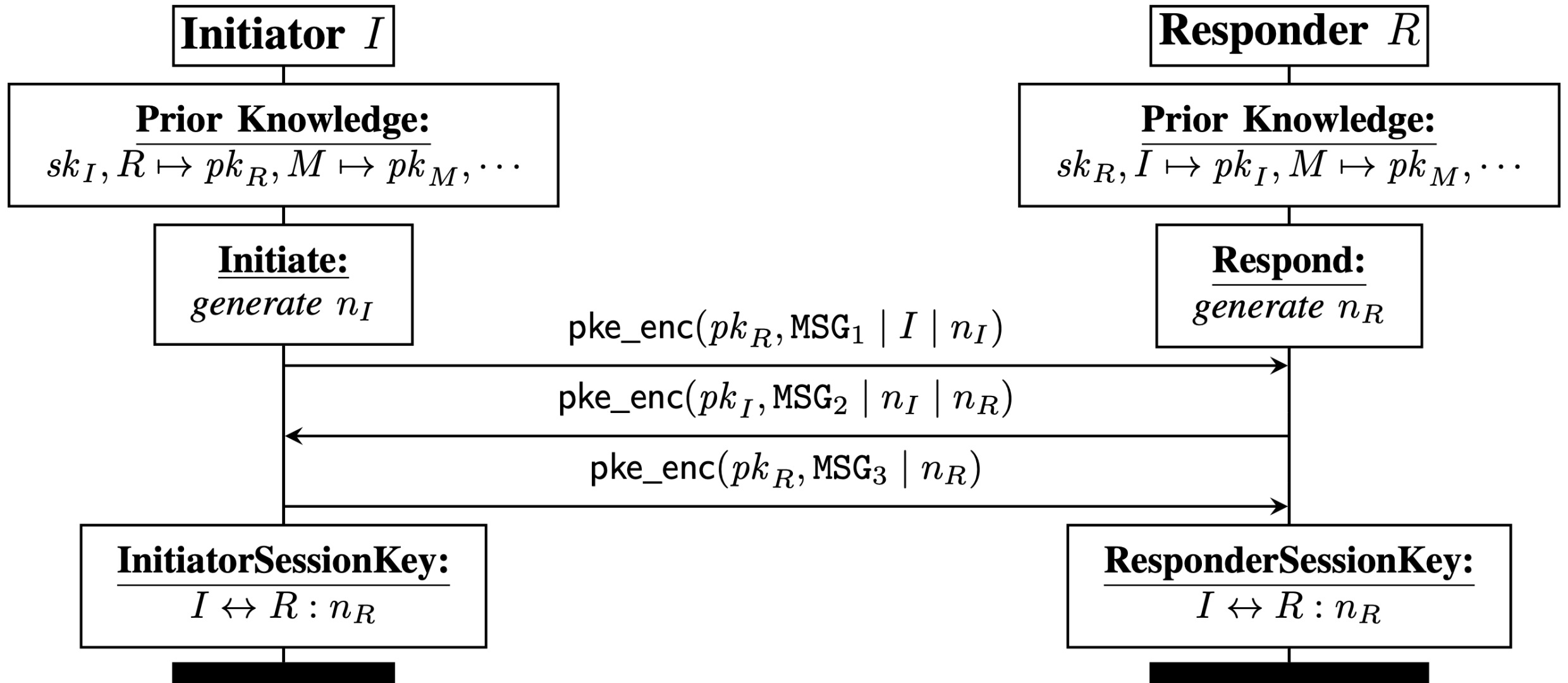- Reason about possible protocol actions

Stating Authentication Goals

```
let protocol_sent p₁ p₂ secret tr = ...
let protocol_received p₁ p₂ secret tr = ...

let authentication_lemma ():
  Lemma (∀ tr p m. (reachable tr ∧
                    protocol_received p₁ p₂ m tr) ⟹
                   (protocol_sent p₁ p₂ m tr ∨
                    compromised p₁ tr))
```
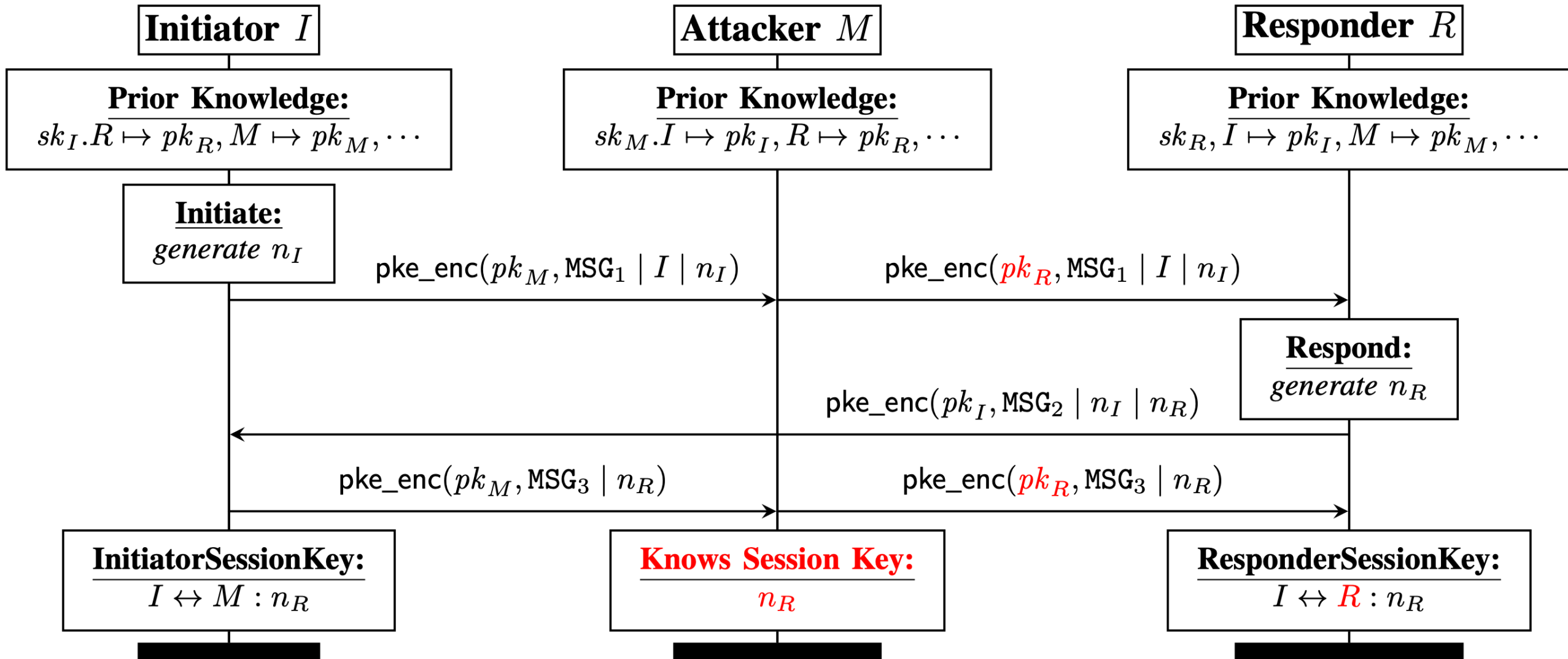
- **Correspondence Assertion:** Received p1 p2 m => Sent p1 p2 m
- Proof by induction on all reachable traces

Modular Labeled Proofs for Crypto Protocols in DY*

Needham-Schroeder Public-Key Protocol

Lowe's Attack on NS-PK

NS-PK in F*: Messages

```
type message =
| Msg1: i:principal → n_i: bytes → message
| Msg2: n_i: bytes → n_r:bytes → message
| Msg3: n_r: bytes → message
val serialize_message: message → bytes
val parse_message: bytes → result message
val parse_message_correctness_lemma: m:message →
   Lemma (parse_message (serialize_message m) == Success m)
```

Precise Message Formats
- serialization and parsing
  with correctness proofs

NS-PK in F*: Session State

```
type session_st =
| SecretKey: secret_key: bytes → session_st
| PublicKey: peer:principal → public_key:bytes → session_st
| ISentMsg1: r:principal → n_i:bytes → session_st
| RSentMsg2: i:principal → n_i:bytes → n_r:bytes → session_st
| ISentMsg3: r:principal → n_i:bytes → n_r:bytes → session_st
| RReceivedMsg3: i:principal → n_r:bytes → session_st
val serialize_session_st: session_st → bytes
val parse_session_st: bytes → result session_st
```

Protocol State Machine

- Stateful protocol code
- Session state storage
- Fine-grained compromise

NS-PK in F*: Protocol Code

```
(* Initiate a new protocol session between send Msg1 *)
let initiate (i r : principal) =
  let pk_r = find_public_key r in
  let n_i = gen (Can_Read [P i; P r]) (PKE_Key "NS") in
  let msg1 = Msg1 i n_i in
  let s_msg1 = serialize_message msg1 in
  let c_msg1 = pke_enc pk_r s_msg1 in
  let st0 = ISentMsg1 r n_i in
  let s_st0 = serialize_session_st st0 in
  let sess_id = new_session_number i in
  new_session i sess_id 0 s_st0;
  log_event i "Initiated" [string_to_bytes r; n_i];
  send i r c_msg1;
  sess_id
(* Process Msg2 and send Msg3 to complete protocol session *)
let initiator_complete (i : principal) (session_id msg_id : nat) =
```

Code for Initiator
- Generates a nonce
- Calls crypto functions
- Stores new session state
- Logs a security ecent
- Sends a message

How do we show this NS-PK implementation is secure?

DY* Verification Architecture
[Euro S&P 2021]

Secrecy Labels for Bytstrings

```
type principal = string
type st_id =
| P: principal → st_id
| S: principal → session:nat → st_id
| V: principal → session:nat → version:nat → st_id
type label =
| Public: label
| Can_Read: list st_id → label
| Meet: label → label → label
| Join: label → label → label
val can_flow: timestamp → label → label → pred
```

Who can read a secret?
- Public: anybody
- CanRead [P a; P b]: a or b

Secrecy Labels for Bytstrings

Meet (Join (Can_Read [P i]) (Can_Read [P r]))
(Meet (Join (Can_Read [V i sid_i 0]) (Can_Read [P r]))
(Meet (Join (Can_Read [V i sid_i 0]) (Can_Read [P r]))
(Join (Can_Read [V i sid_i 0]) (Can_Read [V r sid_r 0]))))

Label for session key in Signal Protocol
- Encodes channel secrecy
- Forward and Post-Compromise security

A Labeled Crypto API

Typed Cryptographic API encodes security assumptions
Using secrecy labels and authentication predicates

```
val pke_enc: #i:nat → #l:label → #s:string →
    public_enc_key i l s →
    m:msg i l{pke_pred m} → msg i Public
val pke_dec: #i:nat → #l:label → #s:string →
    private_dec_key i l s → msg i Public →
    result (m:msg i l{is_publishable i m ∨ pke_pred m})
```

Lowe's Attack as a Type Error

```
(* Process Msg2 and send Msg3 to complete protocol session *)
let initiator_complete (i : principal) (session_id msg_id : nat) =
  let (ver_id,st) = get_session i session_id in
  match parse_session_st st with
  | Success (ISentMsg1 r n_i) →
    let (from,c_msg2) = receive_i i msg_id in
    let sk_i = find_private_key i in
    let pk_r = find_public_key r in
    (match pke_dec sk_i c_msg2 with
    | Success s_msg2 →
      (match parse_message s_msg2 with
      | Success (Msg2 n_i' n_r) →
        if n_i = n_i' then
          let s_msg3 = serialize_message (Msg3 n_r) in
          let c_msg3 = pke_enc pk_r s_msg3 in
          let new_st = ISentMsg3 r n_i n_r in
          let s_new_st = serialize_session_st new_st in
          log_event i "InitiatorDone" [string_to_bytes r; n_i; n_r];
          update_session i session_id ver_id s_new_st;
          send i r c_msg3
        else error "received_incorrect_n_i"
      | _ → error "did_not_receive_a_msg_2")
    | _ → error "decryption_failed")
  | _ → error "incorrect_sesssion_state"
```
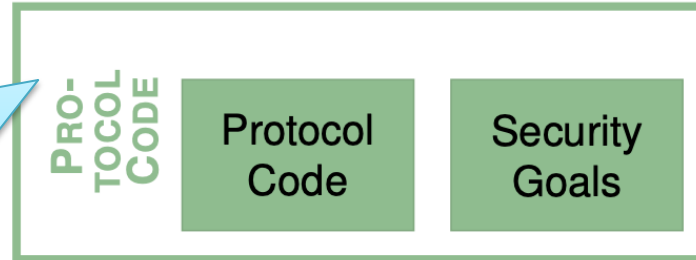
Can n_r be sent to r?

- Does the label of n_r flow to CanRead [P r]?
- Not provable, because Lowe's attack
- Indeed, we can implement and demonstrate symbolic attack in F*

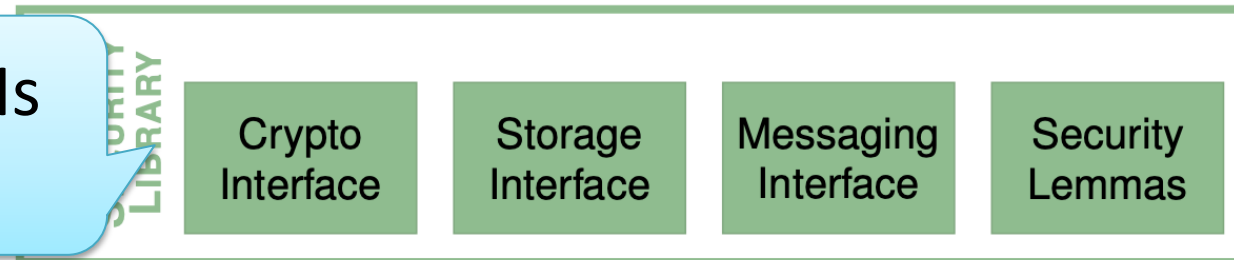# DY* Verification Architecture
[Euro S&P 2021]

DY*: scalable security verification

	Modules	FLoC	PLoC	Verif. Time	Primitives
Generic DY*	9	1,536	1,344	$\approx$ 3.2 min	-
NS-PK	4	439	-	(insecure)	PKE
NSL	5	340	188	$\approx$ 0.5 min	PKE
ISO-DH	5	424	165	$\approx$ 0.9 min	DH, Sig
ISO-KEM	4	426	100	$\approx$ 0.7 min	PKE, Sig
Signal	8	836	719	$\approx$ 1.5 min	DH, Sig, KDF, AEAD, MAC

Proofs require between 50% and 90% annotation overhead
Size of annotation depends on complexity of security goals

Sign-then-Encrypt Protocol

Man-in-the-Middle Attack

Initiator I

Initially Knows:
sk_i, pk_o, pk_r

Attacker O

Initially Knows:
sk_o, pk_i, pk_r

Responder R

Initially Knows:
sk_r, pk_o, pk_i

$\mathsf{penc}(pk_o, \mathsf{sign}(sk_i, 0\|req))$

$\mathsf{penc}(pk_r, \mathsf{sign}(sk_i, 0\|req))$

$\mathsf{penc}(pk_i, \mathsf{sign}(sk_r, 1\|resp))$

Exchange:
$I \leftrightarrow O : req, \perp$

Exchange:
$I \leftrightarrow R : req; resp$

Attacker acting as a valid responder for I,
re-encrypts request to R,
causing an *identity mis-binding attack*

Implementing Sign-Then-Encrypt (demo)

Modeling Computational Assumptions

Modular Type-Based Cryptographic Verification

Sample modular verification (protocol)

RPC protocol using Authenticated Encryption

Bytes

Networking

system libraries

some cryptographic implementation

authenticated encryption

Formatting

message format

security protocols

Secure RPC

RPC API

Adversary Model

any typed F7 program

any typed F# program

active adversaries

application code

Sample modular verification (crypto)

RPC using Encrypt-then-MAC

cryptographic schemes

cryptographic constructions

probabilistic computational indistinguishability

active adversaries

Bytes **Networking**

system libraries

AES-CBC encryption
≈ **IDEAL**
IND-CPA

MAC authentication
≈ **IDEAL**
INT-CMA

Encrypt-then-MAC
authenticated encryption

Formatting
message format

security protocols

Secure RPC
RPC API

Adversary Model

any typed F7 program

any typed F# program

application code

MAC : integrity

Sample functionality:
Message Authentication Codes

```
module MAC

type text = bytes      val macsize
type key  = bytes
type mac  = bytes


val GEN    : unit -> key
val MAC    : key -> text -> mac
val VERIFY: key -> text -> mac -> bool
```

basic F*
interface

This interface says nothing
on the security of MACs.

Message Authentication Codes

MAC keys are abstract

```
module MAC

type text = bytes       val macsize
type key
type mac  = bytes


val GEN   : unit -> key
val MAC   : key -> text -> mac
val VERIFY: key -> text -> mac -> bool
```

Message Authentication Codes

MAC keys are abstract

```
module MAC

type text = bytes        val macsize
type key
type mac  = b:bytes{Length(b)=macsize}


val GEN    : unit -> key
val MAC    : key -> text -> mac
val VERIFY: key -> text -> mac -> bool
```

MACs are fixed sized

Sample functionality:

Message Authentication Codes

ideal F* interface

MAC keys are abstract

```
module MAC

type text = bytes        val macsize
type key
type mac  = b:bytes{Length(b)=macsize}

predicate Msg of key * text

val GEN    : unit -> key
val MAC    : k:key -> t:text{Msg(k,t)} -> mac
val VERIFY: k:key -> t:text -> mac
            -> b:bool{ b=true => Msg(k,t)}
```

MACs are fixed sized

Msg is specified by protocols using MACs

"All verified messages have been MACed"

Sample functionality:

Message Authentication Codes

ideal F* interface

MAC keys are abstract

```
module MAC

type text = bytes        val macsize
type key
type mac  = b:bytes{Length(b)=macsize}

predicate Msg of key * text

val GEN    : unit -> key
val MAC    : k:key -> t:text{Msg(k,t)} -> mac
val VERIFY: k:key -> t:text -> mac
           -> b:bool{ b=true => Msg(k,t)}
```

MACs are fixed sized

Msg is specified by protocols using MACs

"All verified messages have been MACed"

This can't be true! (collisions)

concrete F* implementation (using real crypto)

```
module MAC
open System.Security.Cryptography

let macsize = 20
let GEN()   = randomBytes 16
let MAC k t = (new HASHMACSHA1(k)).ComputeHash t
let VERIFY k t m = (MAC k t = m)
```

Sample computational assumption:

Resistance to Chosen-Message Existential Forgery Attacks (INT-CMA)
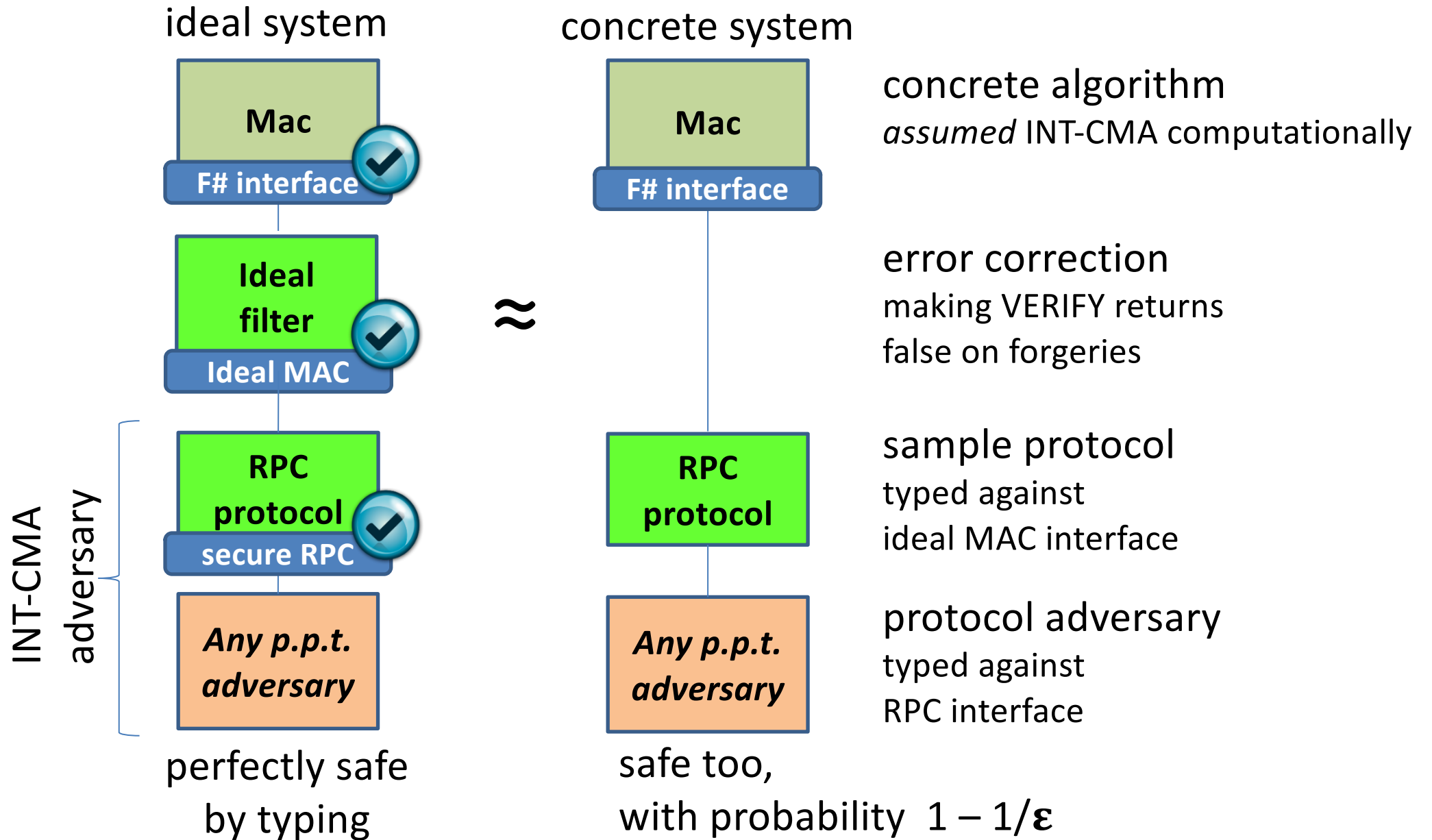
```
module INT_CMA_Game
open Mac

Let private k = GEN()
let private log = ref []
let mac t =
  log := t::!log
  MAC k t
let verify t m =
  let v = VERIFY k t m in
  if v && not (mem t !log) then FORGERY
  v
```

Computational Safety
a probabilistic polytime program calling **mac** and **verify** forges a MAC only with negligible probability [2]

CMA game
(coded in F#)

Computational Safety for MACs

Supporting Key Compromise

ideal F* interface

MAC keys are abstract

MACs are fixed sized

Msg is specified by protocols using MACs

"All verified messages have been MACed"

MAC keys have concrete representations

It is safe to turn keys into bytes when **all messages are verifiable**

```
module MAC

type text = bytes        val macsize
type key
type mac  = b:bytes{Length(b)=macsize}

predicate Msg of key * text

val GEN    : unit -> key
val MAC    : k:key -> t:text{Msg(k,t)} -> mac
val VERIFY: k:key -> t:text -> mac
           -> b:bool{ b=true => Msg(k,t)}


val keysize
type keybytes = b:bytes{Length(b)=keysize}
val LEAK:    k:key{!t. Msg(k,t)} -> b:keybytes
val COERCE: b:keybytes{…}        -> k:key{…}
```

Perfect Secrecy by Typing

- Secrecy is expressed using observational equivalences between systems that differ on their secrets

- We prove (probabilistic, information theoretic) secrecy by typing, relying on type abstraction

$$I_\alpha = \alpha, \ldots, x : T_\alpha, \ldots.$$

P_α range over pure modules such that $\vdash P_\alpha \rightsquigarrow I_\alpha$.

THEOREM   (Secrecy by Typing).
Let A such that $I_\alpha \vdash A : bool$.
For all P_α^0 and P_α^1, we have $P_\alpha^0 \cdot A \approx P_\alpha^1 \cdot A$.

Plaintext Modules

- Encryption is parameterized by a module
  that abstractly define plaintexts, with interface

```
module Plaintext

val size: int
type plain
type repr = b:bytes{Length(b)=size}

val coerce : repr  -> plain // turning bytes into secrets
val leak   : plain -> repr  // breaking secrecy!




val respond: plain -> plain // sample protocol code
```

The size of plaintext is fixed
(as we cannot hide it)

If we remove the **leak** function,
we get secrecy by typing

If we remove the **coerce** function,
we get integrity by typing

Plain may also implement any
protocol functions that operates on secrets

Ideal Interface for Authenticated Encryption

```
module AE
open Plaintext

type key
type cipher = b:bytes{Length(b)= size + 16}

val GEN: unit-> key
val ENC: key -> plain  -> cipher
val DEC: key -> cipher -> plain option
```

- Relying on basic cryptographic assumptions (IND-CPA, INT-CTXT)
  its **ideal implementation** never accesses plaintexts!
  Formally, ideal AE is typed using an abstract **plain** type

  ENC k p   encrypts instead zeros to c & and logs (k,c,p)

  DEC k c   returns Some(p) when (k,c,p) is in the log, or None

An Ideal Interface for CCA2-Secure Encryption

```
module PKENC
open Plain

val pksize: int
type skey
type pkey = b:bytes{ PKey(b) Æ}

val ciphersize: int
type cipher = b:bytes{Length(b)=ciphersize}

val GEN: unit -> pkey * skey
val ENC: pkey -> plain -> cipher
val DEC: skey -> cipher -> plain
```

- Its **ideal implementation** encrypts zeros instead of plaintexts
  so it never accesses plaintext representations,
  and can be typed parametrically

Typed Secrecy from CCA2-Secure Encryption

THEOREM 7 (Asymptotic Secrecy).
Let P^0 and P^1 p.p.t. secret with $\vdash P^b \rightsquigarrow I_{\mathsf{PLAIN}}$.
Let C_{ENC} p.p.t. CCA2-secure with $I^C_{\mathsf{PLAIN}} \vdash C_{\mathsf{ENC}} \rightsquigarrow I^C_{\mathsf{ENC}}$.
Let A p.p.t. with $I_{\mathsf{PLAIN}}, I_{\mathsf{ENC}} \vdash A : bool$.

$$P^0 \cdot C_{\mathsf{ENC}} \cdot A \approx_\varepsilon P^1 \cdot C_{\mathsf{ENC}} \cdot A.$$

THEOREM 8 (Ideal Functionality).
Let P p.p.t. with $\vdash P \rightsquigarrow I^C_{\mathsf{PLAIN}}$ (not necessarily secret)
Let C_{ENC} p.p.t. CCA2-secure with $I^C_{\mathsf{PLAIN}} \vdash C_{\mathsf{ENC}} \rightsquigarrow I^C_{\mathsf{ENC}}$.
Let A p.p.t. with $I^C_{\mathsf{PLAIN}}, I_{\mathsf{ENC}} \vdash A$.

$$P \cdot C_{\mathsf{ENC}} \cdot A \approx_\varepsilon P \cdot C_{\mathsf{ENC}} \cdot F_{\mathsf{ENC}} \cdot A.$$
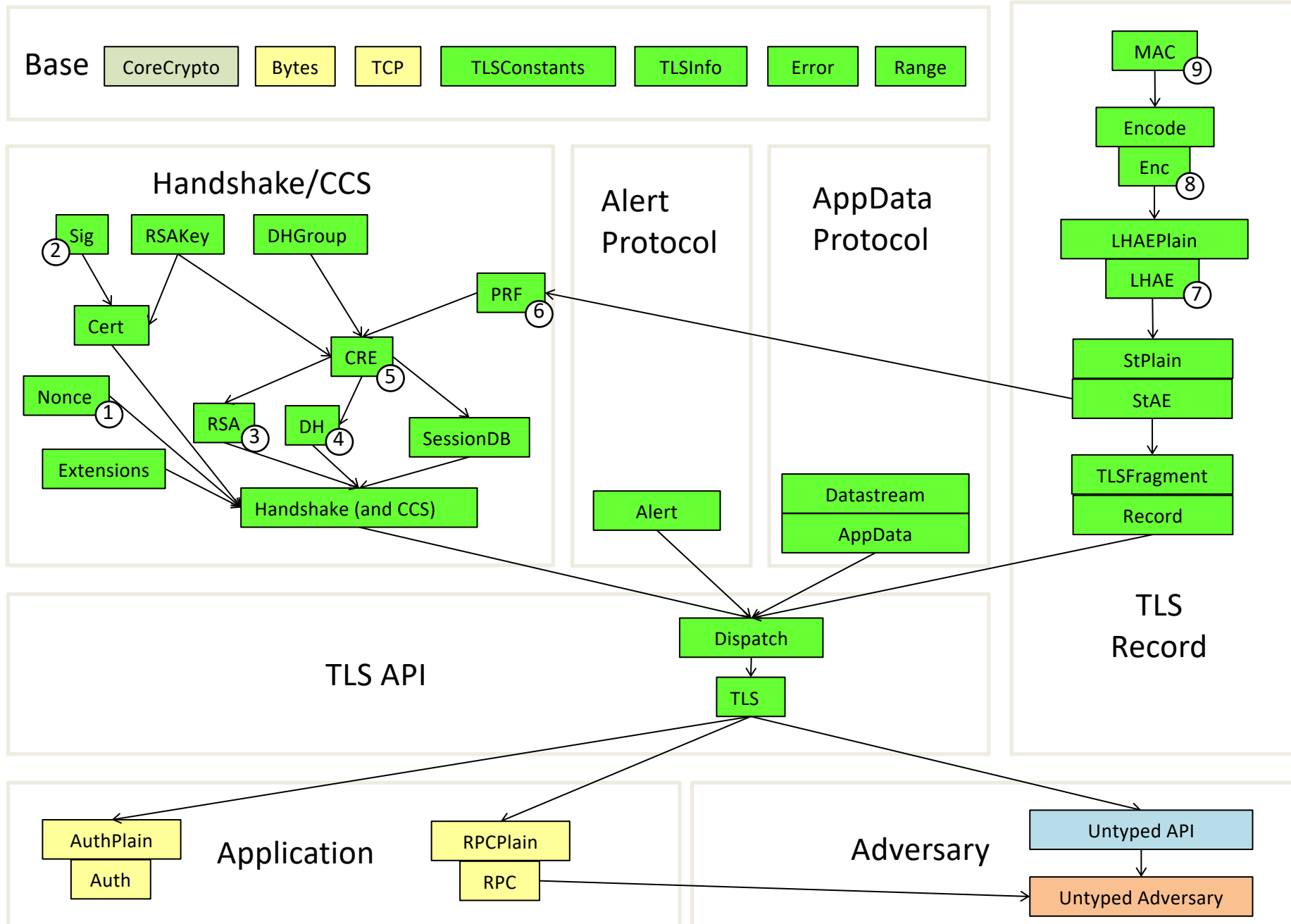
Variants: CPA & Authentication

- With **CPA-secure encryption**, we have a **weaker** ideal interface that demands ciphertext integrity before decryption

```
predicate Encrypted of key * cipher

val ENC: k:key -> plain -> c:cipher{Encrypted(k,c)}
val DEC: k:key -> c:cipher{Encrypted(k,c)} -> plain
```

- With **authenticated encryption**, we have a **stronger** ideal interface that ensure plaintext integrity (much as MACs)

```
predicate Msg of key * plain // defined by protocol

val ENC: k:key -> p:plain{Msg(k,p)} -> cipher
val DEC: k:key -> cipher -> p:plain{Msg(k,p)} option
```

Modular Architecture for miTLS

our main TLS API (outline)

Each application provides its own plaintext module for data streams:

- Typing ensures secrecy and authenticity at safe indexes

Each application creates and runs session & connections in parallel

- Parameters select ciphersuites and certificates
- Results provide detailed information on the protocol state

```
type cn // for each local instance of the protocol

// creating new client and server instances
val connect: TcpStream -> params -> (;Client) nullCn Result
val accept:  TcpStream -> params -> (;Server) nullCn Result

// triggering new handshakes, and closing connections
val rehandshake: c:cn{Role(c)=Client} -> cn Result
val request:     c:cn{Role(c)=Server} -> cn Result
val shutdown:    c:cn -> TcpStream Result

// writing data
type (;c:cn,data:(;c) msg_o) ioresult_o =
| WriteComplete of c':cn
| WritePartial  of c':cn * rest:(;c') msg_o
| MustRead      of c':cn
val write: c:cn -> data:(;c) msg_o -> (;c,data) ioresult_o

// reading data
type (;c:cn) ioresult_i =
| Read      of c':cn * data:(;c) msg_i
| CertQuery of c':cn
| Handshake of c':cn
| Close     of TcpStream
| Warning   of c':cn * a:alertDescription
| Fatal     of a:alertDescription
val read : c:cn -> (;c) ioresult_i
```
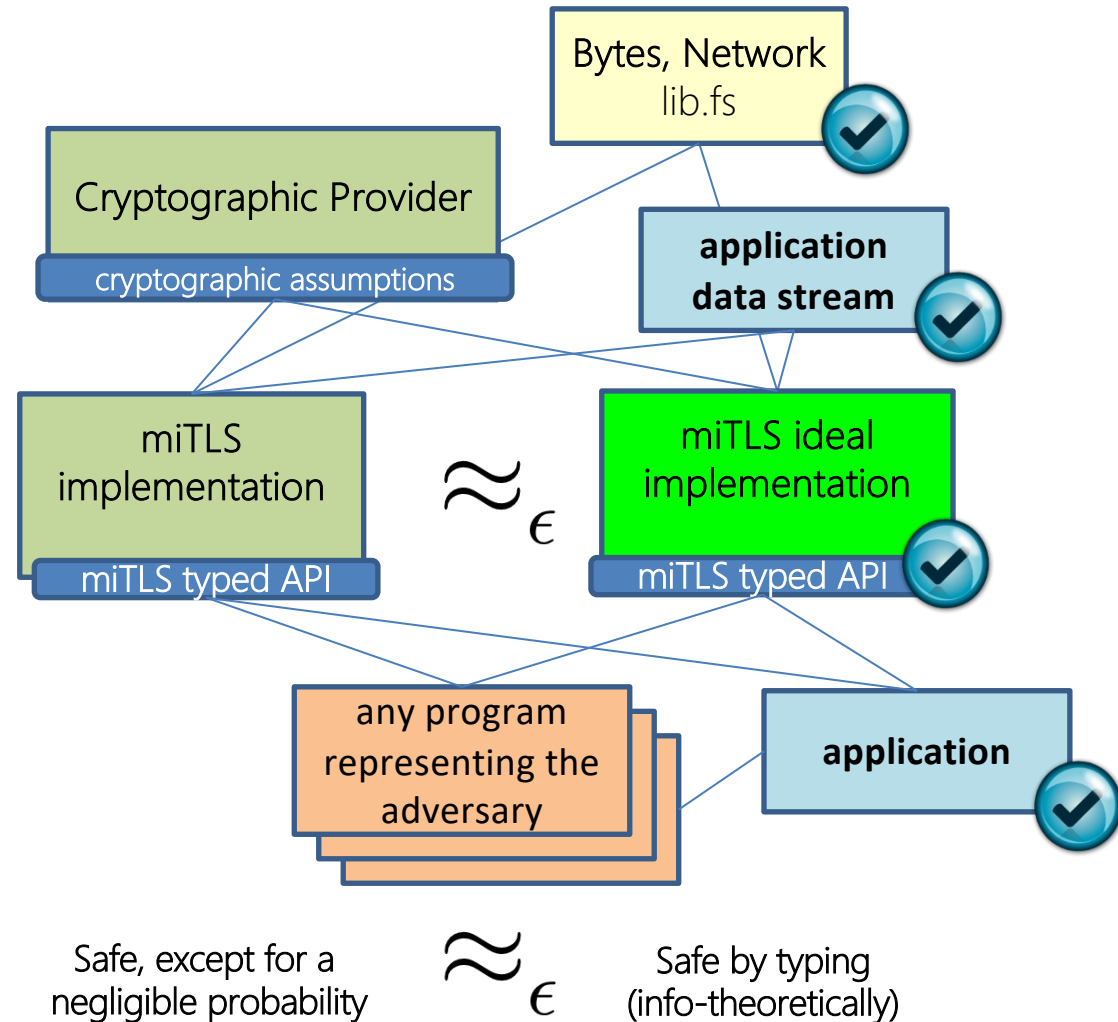
Security theorem

Main crypto result:
concrete TLS & ideal TLS are computationally indistinguishable

We prove that ideal miTLS meets its secure channel specification
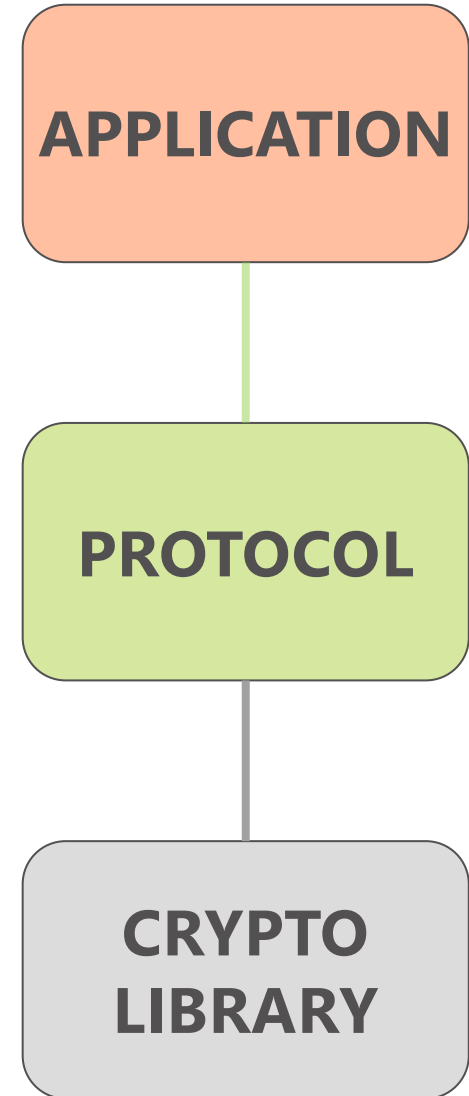using standard program verification (typing)

Safe, except for a negligible probability $\approx_\epsilon$ Safe by typing (info-theoretically)

Final Thoughts

Many pitfalls in cryptographic software

- Need to verify their design+implementation
- Need to verify crypto+protocol+application

Formal security proofs for real-world crypto protocols are now feasible

- TLS 1.3 is an ongoing successful experiment
- Similar results for SSH, Signal, etc.
- Many tools: ProVerif, CryptoVerif, F*, Tamarin, EasyCrypt, VST
- Try them out to build your next proof, or to implement your crypto protocols securely!

APPLICATION

PROTOCOL

CRYPTO LIBRARY

End of Part IV